## Economics 230a, Fall 2013

## Lecture Note 4: Basic Tax Incidence

Tax incidence refers to where the burden of taxation actually falls, as distinguished from who has the legal liability to pay taxes. As with deadweight loss, it is a concept for which the intuition is clear, but for which actual measurement requires the specification of a precise conceptual experiment. It is not enough simply to ask, "what is the incidence of a tax on good x?" We must specify what is done with the revenue, i.e., whether it is (1) spent in a way that has no further effects on welfare (absolute incidence); (2) spent on goods and services, which also have an impact on welfare (balanced-budget incidence); or (3) used to reduce other taxes (differential incidence).

To illustrate the concept of incidence, consider the absolute incidence of a small tax introduced in some competitive market, in which the initial price is $p_{0}$ and the initial quantity $x_{0}$. We introduce a tax, which reduces output, increases the consumer price $q$, and reduces the producer price $p$, in the manner shown below. For simplicity, we will assume that the revenue is spent by the government in the same manner that the consumer would spend it. Thus, total demand (by the consumer and the government) is the same as it would be if the consumer were given the revenue. Starting at an undistorted equilibrium, this is roughly equivalent to compensated demand, since there is no first-order deadweight loss.


The burden of the tax falling on the demand side is the loss of consumer's surplus $\mathrm{A}+\mathrm{B}$, while the burden on the producer is the loss of producer's surplus $\mathrm{C}+\mathrm{D}$, the sum exceeding revenue $(\mathrm{A}+\mathrm{C})$ by the deadweight loss $\mathrm{B}+\mathrm{D}$. For a small change starting at a Pareto optimum, the first-order excess burden is small relative to the revenue cost and we can approximate burdens by $x \Delta q$ for the consumer and $-x \Delta p$ for the producer, with the total burdens equal to revenue in this first-order approximation.

The relative burdens on the demand and supply sides will depend on relative elasticities. Defining the term $\hat{z}=d \log (z)$, and letting the demand and supply elasticities (defined to be nonnegative) be $\eta^{D}$ and $\eta^{S}$, we know that $\hat{x}=-\eta^{D} \hat{q}=\eta^{S} \hat{p}$. Further, if we let $T=(1+\tau)$, where $\tau$ is the ad valorem tax imposed on the producer price, we have $q=T p$, so that $\hat{q}=\hat{T}+\hat{p}$. (Also, assuming that we are starting at a value of $\tau=0, \hat{T}=\tau$.) Thus, setting the two expressions for $\hat{x}$ equal we have $-\eta^{D}(\hat{T}+\hat{p})=\eta^{S} \hat{p} \Rightarrow \hat{p}=\frac{-\eta^{D}}{\eta^{D}+\eta^{S}} \hat{T}$; $\hat{q}=\frac{\eta^{S}}{\eta^{D}+\eta^{S}}$; the ratio of the shares of the burden on consumers and producers is $\eta^{S} / \eta^{D}$, i.e., is proportional to the inverse ratio of the respective elasticities - the greater the responsiveness, the lower the burden.

Note: it does not matter whether the tax is imposed on the buyer or the seller, assuming that prices are flexible.

To analyze incidence more fully, we introduce a simple, two-sector general equilibrium model.

## The Harberger Model

## Assumptions:

- Two factors of production, $K$ and $L$, in fixed overall supply, $\bar{K}$ and $\bar{L}$
- Two competitive sectors of production, $X$ and $Y$, with CRS production functions
- One representative consumer who spends factor income on the two goods
- Starting from an undistorted equilibrium, government raises tax revenue and spends it in exactly the same way the household would

As before, the last assumption implies that for small taxes the changes in total demand will lie along the household's initial indifference curve.

## Basic Equations

By definition,
(1) $\hat{X}-\hat{Y}=-\sigma_{D}\left(\hat{q}_{X}-\hat{q}_{Y}\right)$,
where $\sigma_{D}$ is the demand elasticity of substitution (defined to be non-negative) and $q_{i}$ is the consumer price of good $i$. Also, as a consequence of cost minimization by producers, the derivative of the cost function with respect to the price of a factor is the quantity of that factor used in production; competition implies that price equals marginal cost. It follows that for each production sector $i, \hat{p}_{i}=\theta_{L i} \widehat{w}+\theta_{K i} \hat{r}$, where $w$ and $r$ are the returns to labor and capital and $\theta_{j i}$ is the share of payments to factor $j$ in sector $i$ 's costs. For example, $\theta_{L X}=w L_{X} / p_{X} X$, where $L_{X}$ is the amount of labor used in sector $X$. Note that the shares $\theta$ in each sector must sum to 1 , so that $\hat{p}_{i}=\theta_{L i} \widehat{w}+\left(1-\theta_{L i}\right) \hat{r}$ for each sector. If we subtract this expression for sector $Y$ from that for sector $X$, we get:
(2) $\hat{p}_{X}-\hat{p}_{Y}=\theta^{*}(\widehat{w}-\hat{r})$,
where $\theta^{*}=\left(\theta_{L X}-\theta_{L Y}\right)$ measures the labor intensity of sector $X$ relative to sector $Y$. If $\theta^{*}>0$, the relative price of good $X$ will rise with an increase in the wage relative to the return to capital.

Finally, we can relate factor returns to the production of goods $X$ and $Y$. Intuitively, we would expect an increase in production of good $X$ to lead to greater demand and a higher relative factor return to whichever factor sector $X$ uses more intensively than sector $Y$.

By definition of the production elasticities of substitution, $\sigma_{X}$ and $\sigma_{Y}, \hat{K}_{i}-\hat{L}_{i}=\sigma_{i}(\hat{w}-\hat{r})$ for $i=$ $X, Y$. For convenience, express $K$ and $L$ as ratios of output, e.g., $k_{X} \equiv K_{X} / X$. It follows that
(3) $\quad \hat{k}_{i}-\hat{l}_{i}=\sigma_{i}(\hat{w}-\hat{r}) \quad i=X, Y$.

By the envelope theorem, we know that derivatives of the cost function satisfy $d\left(r k_{i}+w l_{i}\right)=$ $k_{i} d r+l_{i} d w$, so $r d k_{i}+w d l_{i}=0$. This implies that

$$
\begin{equation*}
\left(\frac{r k_{i}}{p_{i}}\right) \hat{k}_{i}+\left(\frac{w l_{i}}{p_{i}}\right) \hat{l}_{i}=\theta_{K i} \hat{k}_{i}+\theta_{L i} \hat{l}_{i}=0, i=X, Y . \tag{4}
\end{equation*}
$$

Finally, note that $L_{X}+L_{Y}=l_{X} X+l_{Y} Y=\bar{L} ; K_{X}+K_{Y}=k_{X} X+k_{Y} Y=\bar{K}$; totally differentiating:
(5a) $\left(\hat{l}_{X}+\hat{X}\right) \lambda_{L X}+\left(\hat{l}_{Y}+\hat{Y}\right) \lambda_{L Y}=0$; also
(5b) $\left(\hat{k}_{X}+\hat{X}\right) \lambda_{K X}+\left(\hat{k}_{Y}+\hat{Y}\right) \lambda_{K Y}=0$
where $\lambda_{L X}=L_{X} / \bar{L}$ is the share of the economy's labor that is used in sector $X$, and the other terms are defined in the same manner.

Now, substitute (4) into (3) for both sectors to get expressions for $\hat{l}_{X}$ and $\hat{l}_{Y}$ and (using the fact that the labor and capital cost shares $\theta$ add to 1 for each sector, and that $\lambda_{L X}+\lambda_{L Y}=1$ ) substitute these expressions into (5a) to obtain:
(6a) $\lambda_{L X} \hat{X}+\lambda_{L Y} \hat{Y}=\left(\lambda_{L X} \theta_{K X} \sigma_{X}+\lambda_{L Y} \theta_{K Y} \sigma_{Y}\right)(\hat{w}-\hat{r})$
Follow the same procedure to get expressions for $\hat{k}_{X}$ and $\hat{k}_{Y}$ to substitute into (5b) to obtain:
(6b) $\lambda_{K X} \hat{X}+\lambda_{K Y} \hat{Y}=-\left(\lambda_{K X} \theta_{L X} \sigma_{X}+\lambda_{K Y} \theta_{L Y} \sigma_{Y}\right)(\hat{w}-\hat{r})$,
and subtract (6b) from (6a) to obtain:

$$
\begin{equation*}
\lambda^{*}(\hat{X}-\hat{Y})=\left(a_{X} \sigma_{X}+a_{Y} \sigma_{Y}\right)(\hat{w}-\hat{r})=\bar{\sigma}(\hat{w}-\hat{r}) \tag{7}
\end{equation*}
$$

where $a_{i}\left(=\lambda_{L i} \theta_{K i}+\lambda_{K i} \theta_{L i}\right)$ is a weighted average of sector $i$ 's share of production, as measured by its use of labor and capital, $\lambda_{K i}$, and labor, $\lambda_{L i}$, and $\lambda^{*}\left(=\lambda_{L X}-\lambda_{K X}\right.$ ) is positive (negative) if sector $X$ is more (less) labor intensive than sector $Y$. As expected, a shift in production toward $X$ will increase the relative return to the factor that $X$ uses relatively intensively. The effect will be stronger the smaller is the average elasticity of substitution, $\bar{\sigma}$, because it will take larger changes in factor prices to induce the changes in factor intensities needed to clear factor markets.

Note that (2) and (7) combined provide an expression for the production possibilities frontier,
(8) $\quad \hat{p}_{X}-\hat{p}_{Y}=\frac{\lambda^{*} \theta^{*}}{\bar{\sigma}}(\hat{X}-\hat{Y})$. (Note that $\operatorname{sgn}\left(\lambda^{*}\right)=\operatorname{sgn}\left(\theta^{*}\right)$, so the frontier is convex.)

Equations (1), (2), and (7) are a system in four unknowns, $(\widehat{w}-\hat{r}),\left(\hat{p}_{X}-\hat{p}_{Y}\right),(\hat{X}-Y)$ and $\left(\hat{q}_{X}-\hat{q}_{Y}\right)$. We add a fourth equation by introducing a tax. We begin with a tax on good $X$, setting $q_{X}=T_{X} p_{X}$, so that:

$$
\begin{equation*}
\hat{q}_{X}-\hat{q}_{Y}=\hat{p}_{X}+\hat{T}_{X}-\hat{p}_{Y} \tag{9}
\end{equation*}
$$

Solving this system of equations, we obtain:

$$
\begin{equation*}
\hat{p}_{X}-\hat{p}_{Y}=-\frac{\sigma_{D}}{\frac{\bar{\sigma}}{\pi^{*} \theta^{*}}+\sigma_{D}} \hat{T}_{x} ; \quad \text { and } \quad \text { (11) } \quad \hat{q}_{X}-\hat{p}_{Y}=\frac{\frac{\bar{\sigma}}{\lambda^{*} \theta^{*}}}{\frac{\overline{\lambda^{*}} \theta^{*}}{}+\sigma_{D}} \hat{T}_{x} \tag{10}
\end{equation*}
$$

Expressions (10) and (11) say that, if we take good $Y$ as the numeraire (i.e., $\hat{p}_{Y}=0$ ), the burden of the tax is borne on the demand and supply sides of $X$ according to the values of terms that relate to demand and supply. These expressions are basically equivalent to those derived in the simple partial equilibrium example based on demand and supply elasticities. Note that the term $\frac{\bar{\sigma}}{\lambda^{*} \theta^{*}}$ comes from the expression for the production possibilities frontier, (8). Under profit
maximization, $p_{x} d X+p_{Y} d Y=0 \Rightarrow \hat{Y}=-\frac{p_{\chi} X}{p_{Y} Y} \hat{X}$, so (8) implies:
(8') $\quad \hat{X}\left(1+\frac{p_{x} X}{p_{Y} Y}\right)=\frac{\bar{\sigma}}{\lambda^{*} \theta^{*}}\left(\hat{p}_{x}-\hat{p}_{y}\right)$
With good $Y$ as numeraire, $\hat{p}_{y}=0$ and ( $8^{\prime}$ ) may be rewritten:

$$
\begin{equation*}
\frac{\bar{\sigma}}{\lambda^{*} \theta^{*}}=\frac{\hat{X}}{\hat{p}_{x}}\left(1+\frac{p_{x} X}{p_{Y} Y}\right)=\eta_{X}^{s}\left(1+\frac{p_{x} X}{p_{Y} Y}\right), \tag{12}
\end{equation*}
$$

where $\eta_{X}^{S}$ is the elasticity of supply of good $X$ with respect to its producer price. Now, consider consumer demand, which is determined by the elasticity of substitution, $\sigma_{D}$, according to (1).
Under utility maximization, $d U=q_{X} d X+p_{Y} d Y=0 \Rightarrow \hat{Y}=-\frac{q_{X} X}{p_{Y} Y} \hat{X}$, so (1) implies:

$$
\hat{X}\left(1+\frac{q_{x} X}{p_{Y} Y}\right)=-\sigma_{D}\left(\hat{q}_{x}-\hat{p}_{y}\right)
$$

Again using the fact that good $Y$ is numeraire, (1') may be rewritten:

$$
\begin{equation*}
\sigma_{D}=-\frac{\hat{X}}{\hat{q}_{X}}\left(1+\frac{q_{X} X}{p_{Y} Y}\right)=\eta_{X}^{D}\left(1+\frac{q_{X} X}{p_{Y} Y}\right) \tag{13}
\end{equation*}
$$

where $\eta_{X}^{D}$ is the elasticity of demand of good $X$ with respect to its consumer price. Substituting (12) and (13) into the incidence expression (11), and noting that $q_{X}=p_{X}$ in the initial equilibrium, we have:

$$
\begin{equation*}
\hat{q}_{x}-\hat{p}_{y}=\frac{\eta_{X}^{S}}{\eta_{X}^{D}+\eta_{X}^{D}} \hat{T}_{x} \tag{14}
\end{equation*}
$$

which is precisely the partial-equilibrium expression for the impact on the taxed good's consumer price.

Returning to the general incidence solution, we combine (10) and (2) to obtain:

$$
\begin{equation*}
(\widehat{w}-\hat{r})=-\frac{1}{\theta^{*}} \frac{\sigma_{D}}{\lambda^{*} \theta^{*}}+\sigma_{D} \widehat{T}_{x} . \tag{15}
\end{equation*}
$$

This expression says that the tax on good $X$, which lowers the producer price of good $X$, will also lower the ratio $w / r$ if sector $X$ is labor intensive - a tax on the labor-intensive good is relatively bad for labor. How would we measure the share of the burden borne by labor? Intuitively, if $w / r$ is fixed, i.e., $\widehat{w}-\hat{r}=0$, then the tax is borne in proportion to each factor's share of income since relative rates of return don't change, and factor supplies are fixed, an increase in the consumer price of good $X$ will lower real factor incomes of labor and capital by the same proportion. More generally, we can ask what fraction, $\psi$, of the tax revenue we would have to give back to labor in order to keep labor's share of gross income (including the tax), $\frac{w L+\psi\left(T_{X}-1\right) p_{X} X}{w L+r K+\left(T_{X}-1\right) p_{X} X}$, constant. Clearly, if $w / r$ doesn't change as the tax is imposed, $\psi=\frac{w L}{w L+r K}$. If $\widehat{w}-\hat{r}<(>) 0, \psi$ is larger (smaller).

Now, consider a partial factor tax on capital used in sector $X$, which is how Harberger conceived of the corporate income tax - as an additional tax on capital used in the corporate sector. (Note that a general tax on capital income in this model is simply borne by capital, as capital is in fixed overall supply, so the only interesting factor-tax incidence question involves the differential tax in one sector.) Intuitively, we should expect this tax to have two effects. The first will be to raise the cost of good $X$, just like the excise tax. (The fact that the tax is levied on the production side, rather than on the transaction with the consumer, is irrelevant.) The second will be to discourage the use of capital in production, which should shift the incidence further onto capital. These are sometimes referred to as the excise tax effect and the factor substitution effect.

To solve for the effects of this tax, we replace $r$ with $r T_{K X}$ in any equations involving the return to capital in sector $X$. Thus, we get $\hat{p}_{X}=\theta_{L X} \widehat{w}+\theta_{K X}\left(\hat{r}+\widehat{T}_{K X}\right)$, which implies:
(2') $\quad \hat{p}_{X}-\hat{p}_{Y}=\theta^{*}(\widehat{w}-\hat{r})+\theta_{K X} \hat{T}_{K X}$
This expression picks up the excise tax effect. Also, equation (7) is modified as follows:

$$
\begin{equation*}
\lambda^{*}(\hat{X}-\hat{Y})=a_{X} \sigma_{X}\left(\hat{w}-\hat{r}-\hat{T}_{K X}\right)+a_{Y} \sigma_{Y}(\hat{w}-\hat{r})=\bar{\sigma}(\hat{w}-\hat{r})-a_{X} \sigma_{X} \hat{T}_{K X}, \tag{7'}
\end{equation*}
$$

which picks up the factor substitution effect, showing, for example, that even if $X / Y$ doesn't change, $\widehat{w}-\hat{r}>0$.

Solving (1), (2'), and ( $7^{\prime}$ ) (and using the fact that consumer prices $q$ and producer prices $p$ are equal - the tax is imposed on producers), we get the analogue for (15) above:
(15') $\quad(\widehat{w}-\hat{r})=\frac{-\frac{1}{\theta^{*}} \sigma_{D} \theta_{K X}+\frac{a_{X} \sigma_{X}}{\lambda^{*} \theta^{*}}}{\frac{\bar{\sigma}}{\lambda^{*} \theta^{*}}+\sigma_{D}} \widehat{T}_{K x}$,
in which the two terms in the numerator of the right-hand side account for the excise tax effect (which can be positive or negative) and the factor substitution effect (which is non-negative).

Harberger showed that under a variety of reasonable assumptions (such as all three elasticities being equal), capital bears exactly 100 percent of the tax. Note that this is the burden on all capital - as capital flees the corporate sector, it depresses returns in the noncorporate sector as well. Both the realism of the model and the characterization of the corporate income tax as an extra tax on capital in the corporate sector are subject to question, as we shall discuss in subsequent lectures.

## Further Issues

The analysis so far assumes that individual decisions are rational and fully informed. In some cases, we may wish to modify these assumptions.

What if taxes vary in their salience? Standard results may require modification. For example, it may no longer be the case that taxes have the same incidence whether they are imposed on buyer or seller, if the tax imposed on the seller is already included in the price while the tax on the buyer is added on after the price is quoted. The degree of salience affects both incidence and efficiency. If an individual does not recognize a tax, this is equivalent to having a lower demand elasticity, which increases incidence on the buyer. Also, it would seem to reduce deadweight loss, but there is a second, offsetting factor here, that by not perceiving a tax, the buyer will than end up with lower income than expected and hence have to cut back on other purchases, rather than spreading the income loss among all purchases. See the paper by Chetty for a discussion of these issues.

A second issue involves the incidence of taxes when individuals may have self-control problems. This is particularly relevant for "sin" taxes on items like alcohol and tobacco. Standard incidence analysis of a tobacco tax, for example, would suggest that the burden of increased prices is very regressive, because tobacco demand has a very low income elasticity. But, what if, by some measure, individuals are making mistakes regarding whether and how much to smoke? Then, if a tobacco tax reduces demand, it may be correcting an "internality," which in itself would make smokers better off. Of course, they would still be paying a higher price for the tobacco they buy, and the changes in demand patterns would affect the equilibrium and further affect incidence. See the paper by Gruber and Kőszegi.

